Low Temperature Physics/Fizyka Nyzkykh Temperatur, 2022, Vol. 48, No. 12, pp. 1187–1193

On the new universality class in structurally disordered n-vector model with long-range interactions

Dmytro Shapoval[1,2], Maxym Dudka[1,2], and Yurij Holovatch[1,2,3]

[1] *Institute of Condensed Matter Physics of the National Academy of Sciences of Ukraine Lviv 79011, Ukraine*

[2] $\mathbb{L}^4$ *Collaboration & Doctoral College for the Statistical Physics of Complex Systems Leipzig–Lorraine–Lviv–Coventry, Europe*

[3] *Centre for Fluid and Complex Systems, Coventry University, Coventry, CV1 5FB, United Kingdom*

E-mail: hol@icmp.lviv.ua

Received August 15, 2022, published online October 24, 2022

We study a stability boundary of a region where nontrivial critical behavior of an n-vector model with long-range power-law decaying interactions is induced by the presence of a structural disorder (e.g., weak quenched dilution). This boundary is given by the marginal dimension of the order parameter n_c dependent on space dimension, d, and a control parameter of the interaction decay, σ, below which the model belongs to the new dilution-induced universality class. Exploiting the Harris criterion and recent field theoretical renormalization group results for the pure model with long-range interactions, we get n_c as a three loop $\varepsilon = 2\sigma - d$-expansion. We provide numerical values for n_c applying series resummation methods. Our results show that not only the Ising systems ($n = 1$) can belong to the new disorder-induced long-range universality class at $d = 2$ and 3.

Keywords: long-range interaction, quenched disorder, renormalization group, marginal dimension.

1. Introduction

Year 2022 marks the 110th birth anniversary of Oleksandr (a.k.a. A.S.) Davydov, an outstanding physicist, known for his seminal contributions in the fields of solid state theory, nuclear physics and biophysics, for many years he served as a director of the Bogolyubov Institute for Theoretical Physics in Kyiv. The authors of this paper who studied physics also from O. Davydov's books [1–3] consider as a great honor to contribute to the Festschrift prepared on this occasion. In our paper, we use the perturbative field theoretical renormalization approach refined by the resummation of asymptotic series expansions to study universal features of criticality. Although such problems were beyond the focus of attention of O. Davydov, the concepts called for their analysis: Symmetry, Space dimension, Range of interaction belong to the central ones in physics. In the paper, we show how their interplay defines universal features of one of the key models currently used to understand qualitatively and to describe quantitatively the critical behavior in condensed matter and beyond. Therefore, conceptually the results presented in this paper are related to those discussed in O. Davydov's seminal works. For this reason, we have chosen to present these results here.

Since interparticle forces in various physical, chemical, and biological systems are often of a long-range nature, models with long-range interaction attract much attention. They have found their applications in studies of gravitational, dipolar, cold Coulomb systems, problems in plasma, atomic and nuclear physics, hydrodynamics and geophysical fluid mechanics (see [4–6] and references therein). Systems with long-range interactions possess properties that differ from those with short-range interactions. To give an example, even weak long-range interactions effectively modify the critical properties and may induce the long-range order in one-dimensional systems [5, 6].

In this paper, we will discuss possible changes in the critical behavior of a many-particle system caused by mutual effects of long-range interactions and structural disorder. To this end, we will consider the now standard n-vector spin model with the Hamiltonian

$$\mathcal{H} = -\frac{1}{2}\sum_{\mathbf{x},\mathbf{x}'} J(|\,\mathbf{x}-\mathbf{x}'\,|)\,\mathbf{S}_{\mathbf{x}}\mathbf{S}_{\mathbf{x}'} \qquad (1)$$

that describes a system of classical n-component vectors ("spins") $\mathbf{S}_{\mathbf{x}} = (S^1_{\mathbf{x}}, S^2_{\mathbf{x}},\ldots,S^n_{\mathbf{x}},)$ located at sites $\mathbf{x}$ of a d-dimensional lattice and interacting via the distance-dependent potential $J(x)$. The r.h.s of Eq. (1) contains a scalar product of spins and the sums over $\mathbf{x}, \mathbf{x}'$ span all lattice sites. Influence of long-range interactions on the critical behavior is usually exemplified by the power-law decaying interaction:

$$J(x) \sim x^{-d-\sigma}, \tag{2}$$

where $\sigma > 0$ is a control parameter of the interaction decay.

As we discuss in more details below, in the case of a regular (nondisordered) lattices, the critical behavior of the model (1) is governed by the triple of parameters (d, n, σ): depending on their values, the model may manifest a low-temperature long-range order that emerges as a second-order phase transition. As long as the model Hamiltonian (1) is formulated in terms of elementary magnets — "spins", the long-range ordered phase is usually associated with an emergence of spontaneous magnetization. We will use such magnetic terminology too, however, let us note that the model itself as well as our discussion without the loss of generality concern much more wide range of types of ordering [7, 8] in physics and beyond. The transition to the ordered "magnetic" phase is characterized by certain universal (i.e., independent on specific system details) features. It is said to belong to a certain universality class. Systems that belong to the same universality class share the values of critical exponents, amplitude ratios, scaling functions. Our goal in this paper is to show how these universal features are changed if instead of a regular lattice structure, it is cosidered the disordered one. Disorder in the lattice structure may be imposed, e.g., by dilution, when a part of the lattice sites in (1) are not occupied by spins. Such situation mimics randomness and nonregularities that are so often met in nature and attract much interest in modern theory of critical phenomena (see, e.g., [8] and references therein). To quantify our analysis, we will calculate the marginal dimension $n_c(\sigma)$: for given space dimension d it discriminates between different universality classes. The rest of the paper is organized as follows. In the next section, we give a short review of the results present so far, in Sec. 3 we describe the field-theoretical renormalization group picture of the critical behavior for the long-range interacting n-vector model with disorder. The results for n_c are given in Sec. 4, and we summarize our study in Sec. 5. Some lengthy expressions are given in the Appendix.

2. Review

In this section, we briefly review some results relevant for our analysis. To proceed further, we explain terminology used throughout the paper. The n-vector model (1) with short-range interactions — since the corresponding free energy is invariant with respect to rotations in the n-dimensional magnetization space it is also called the $O(n)$-symmetric model — manifests the second-order phase transition for the lattice space dimension $d > d_{lc}$. The lower critical dimension $d_{lc} = 1$ for the discrete (Ising) case $n = 1$, whereas $d_{lc} = 2$ for $n > 1$. Critical exponents and other universal properties of the short-range n-vector model depend on n and d in a nontrivial way in the region $d_{lc} < d \le d_{uc}$. They are said to belong to the short-range universality class. For d larger than the upper critical dimension $d_{uc} = 4$, the model is governed by the mean-field exponents, see [9] for more details. Introducing the long-range interaction (2) drastically changes the picture of the critical behavior of the n-vector model (1). Calculations performed for the three-dimensional spherical model [10] (it corresponds to the n-vector model at $d = 3$, $n = \infty$) show that for $\sigma > 2$ the critical properties are governed by the short-range critical exponents, while for $\sigma < 2$ one has two regimes depending on the value of σ: with mean-field critical exponents and with the σ-dependent ones. One-dimensional Ising model ($d = 1$, $n = 1$) with interaction (2) was proven to have phase transition to the long-range-ordered phase at nonzero temperature [11]. Field-theoretical renormalization group (RG) analysis of the long-range interacting n-vector model gives three universality classes in dependence on σ [12]: (i) the mean-field critical behavior for $\sigma \le d/2$, (ii) *the short-range universality class* for $\sigma \ge 2$ with critical exponents coinciding with those of the model with short-range interactions, (iii) *the long-range universality class* for $d/2 < \sigma < 2$, where critical exponents depend on σ. Later it was established that the actual boundary between the short-range and the long-range universality classes lies at $\sigma = 2 - \eta_{SR}$ [13, 14] rather than at $\sigma = 2$, η_{SR} is the pair correlation function critical exponent of the short-range model. Such picture was corroborated by other approaches including nonperturbative variant of RG (NPRG) [15], Monte Carlo simulations [16] and conformal bootstrap [17] (for other references and discussion see the review [18]).

Another factor discussed in this paper is the structural disorder and its impact on the critical behavior. The influence of structural inhomogeneity on the universal properties of physical systems continues to be a hot research topic both for academic and practical reasons, since almost all materials are characterized by a certain degree of disorder in their structure. Structural inhomogeneities in magnetic systems are of different nature, which in turn may lead to different changes in critical behavior. In the case of strong structural disorder, randomness is accompanied by frustration and percolation effects and often leads to absence of the long-range magnetic order. The case of weak structural disorder is not that obvious. Here we focus specifically on presence of the weak quenched disorder in lattice structure, which may be implemented into model (1) via dilution by point-like uncorrelated (or short-range correlated) quenched nonmagnetic inhmogeneities. Its relevance for the critical behavior is given by the Harris criterion [19]. The criterion states that the structural disorder (quenched dilution) leads to a new universality class of the magnetic phase transition only if the heat capacity critical exponent of the undiluted (pure) system is positive, $\alpha_p > 0$, i.e., if the heat capacity of the pure system diverges at the critical point. Correspondingly, the disorder is irrelevant if $\alpha_p < 0$. For the short-range n-vector model at $d = 3$, $\alpha_p > 0$ for $n = 1$ (Ising model), whereas $\alpha_p < 0$ for $n \geq 2$, therefore, due to the Harris criterion the diluted n-vector model at $n \geq 2$ shares the universal properties of its undiluted counterpart [20, 21]. Unlike the short-range n-vector model, the long-range one manifests the new universality class induced by dilution also in the region $n \geq 2$, as was shown in the RG study of Ref. 22 within two-loop approximation. This result was also corroborated by the low-temperature RG [23]. However, the estimates of the regions of values (d, n, σ), where the new disordered long-range universality class governs the critical behavior were not satisfactory [22]. The perturbative RG expansions having zero radius of convergence, additional resummation procedures have to be applied in order to get reliable numerical data on their basis [7]. Especially it concerns the Ising model ($n = 1$), where the degeneracy of the RG equations (similarly as in the short-range case [20, 21]) makes the expansion parameter to be $\sqrt{\varepsilon}$ [24]. Reliable results for the last case were obtained within a massive renormalization scheme with resummation of the RG functions [25].

A remarkable feature of the Harris criterion is that it allows to forecast structural-disorder-induced changes in the universality class of a pure system without explicit calculation of the RG functions for the diluted one. Indeed, if the structural disorder changes the universality class only when the heat capacity of the pure system diverges (i.e., when $\alpha_p > 0$), one can use the condition $\alpha_p = 0$, as an equation to define parameters (d, n, σ) that discriminate between different universality classes. For given space dimension d, such equation defines the so-called marginal order-parameter dimension $n_c(\sigma)$. Similar to critical exponents and critical amplitude ratios, the marginal dimensions are universal quantities, reachable in experiments and numerical simulations and are the subject of intensive studies [26–31]. In the next sections, we will calculate the marginal dimension $n_c(\sigma)$ for the diluted long-range n-vector model at space dimensions $d = 2$ and 3. This marginal dimension line in the (n, σ) parametric plane defines a boundary of stability between the pure long-range and the disordered long-range universality classes. To this end, we use the recent three-loop RG results for the critical exponents of the pure n-vector model with long-range power-law decaying interactions [32].

3. Field-theoretical renormalization group description

The progress achieved in qualitative understanding and quantitative description of critical phenomena to a large extend is due to application of RG methods [7]. In the field-theoretical RG approach, the critical properties of the n-vector model (1) with the power-law decaying long-range interactions (2) are described by analyzing the effective Hamiltonian [33]:

$$\mathcal{H} = \int d^d x \left\{ \frac{1}{2}\left((\nabla^{\sigma/2}\varphi)^2 + r_0\varphi^2 \right) + \frac{u_0}{4!}(\varphi^2)^2 \right\}, \qquad (3)$$

where $\varphi = \varphi(\mathbf{x}) = \{\varphi_1(\mathbf{x}), \ldots, \varphi_n(\mathbf{x})\}$ is an n-component vector field, u_0 is the unrenormalized coupling, r_0 defines the temperature distance to the critical point, and $\nabla^{\sigma/2}$ is a symbolic notation for the fractional derivative. The last is defined via its action in the momentum space and leads to the propagator term q^σ rather than q^2, as in the case of short-range interactions. Power counting gives the upper critical dimension $d_{uc} = 2\sigma$, which at $\sigma = 2$ coincides with the traditional $d_{uc} = 4$. The effective Hamiltonian (3) is relevant for the case $0 < \sigma < 2 - \eta_{SR}$. To study crossover to the short-range case $\sigma \to 2$, one should include the traditional term $(\nabla\varphi)^2$ into (3).

In the field-theoretical RG approach, a critical point corresponds to a reachable and stable fixed point (FP) of the RG transformation. It has been found [12] that the nontrivial FP determining new long-range universality class is stable for $d/2 < \sigma$. Critical exponents within this universality class were calculated in $\varepsilon = 2\sigma - d$-expansion up to order ε^2 [12, 33, 34] and up to order $1/n$ [35] in $1/n$-expansion. Estimates for the critical exponents of three-dimensional systems were obtained within the massive renormalization approach completed by resummation in two-loop approximation [36]. The RG results in three-loop approximation were obtained only recently [32]. Monte Carlo estimates for the critical exponents in this universality class have been obtained only for the $n = 1$ Ising case, mainly in one and two space dimensions (for collection of references, see [32]). Only a few Monte Carlo results are available for the three-dimensional case [37].

The presence of uncorrelated nonmagnetic impurities (weak quenched structural disorder) is usually modeled by fluctuations of the local phase transition temperature [38]. Introducing $\phi = \phi(\mathbf{x})$ as the field of local critical temperature fluctuations, one obtains the following effective Hamiltonian for the structurally disordered system:

$$\mathcal{H} = \int d^d x \left\{ \frac{1}{2}\left((\nabla^{\sigma/2}\varphi)^2 + (r_0 + \phi)\varphi^2 \right) + \frac{u_0}{4!}(\varphi^2)^2 \right\}, \qquad (4)$$

where the random variable ϕ has a Gaussian distribution with zero mean and a correlator containing the second coupling v_0:

$$\langle \phi(\mathbf{x}) \rangle = 0, \; \langle \phi(\mathbf{x})\phi(\mathbf{x}') \rangle = v_0 \delta(\mathbf{x} - \mathbf{x}'). \qquad (5)$$

The angular brackets $\langle \ldots \rangle$ indicate an average over the random variable ϕ distribution.

The RG picture for the disordered long-range model (4) is similar to that for its short-range analog [20, 21]. In the parametric space of couplings (u, v), the critical properties of the model (4) are governed by four FPs (u^*, v^*) in dependence on values $\varepsilon = 2\sigma - d$ and n [22, 24]: Gaussian FP $(u^* = 0,\ v^* = 0)$, nonphysical FP $(u^* = 0,\ v^* \neq 0)$, Heisenberg long-range or pure long-range FP $(u^* = 0,\ v^* \neq 0)$, and disordered long-range FP $(u^* \neq 0,\ v^* \neq 0)$. The Gaussian FP is always unstable below critical dimension $d_u = 2\sigma$ $(\varepsilon > 0)$, while the nonphysical FP is always stable in this case, however, it is not accessible from initial conditions appropriate for the model described by (4) and (5). For $\varepsilon > 0$ and $n > n_c$, the long-range Heisenberg FP is stable and the disordered one is unstable, while for $n < n_c$ the FPs swap their stability. Therefore, for $n > n_c$ the universal critical exponents of the diluted model (4) coincide with those of the model (3). For $n < n_c$, model (4) belongs to the new disorder-induced long-range universality class. The boundary between these two regimes is determined by the marginal dimension $n_c(d, \sigma)$.

So far, the value of n_c for the long-range n-vector model was known in the framework of the two-loop approximation [39], which gave the result

$$n_c = 4 - 4[\psi(1) - 2\psi(\sigma/2) + \psi(\sigma)]\varepsilon + O(\varepsilon^2), \qquad (6)$$

where $\psi(x)$ is the digamma function. The asymptotic nature of this series together with its shortness made it difficult to get reliable numerical estimates on its basis. In the next section, we will proceed getting the next order of the ε-expansion and delivering numerical estimates for $n_c(\sigma)$ at certain space dimensions with the help of resummation procedures.

4. Calculation of the marginal dimension

As mentioned above, the marginal dimension n_c of a weakly diluted n-vector model with power-law decaying interactions can be obtained on the base of the critical exponents for the undiluted model. As a consequence of the Harris criterion, the master equation for determining n_c is

$$\alpha_p(n_c, d, \sigma) = 0. \qquad (7)$$

The treatment of Eq. (7) by means of the field-theoretical RG approach can be performed in various schemes. Here we exploit the results of dimensional regularization with the minimal subtraction [40], allowing to obtain quantities of interest by familiar ε-expansion with $\varepsilon = 2\sigma - d$ in our case. To get $n_c(d, \sigma)$, we use the hyperscaling relation $\alpha_p = 2 - d\nu_p$ and ε-expansion for the critical exponent ν_p of the n-vector model with long-range interactions, which is known in the three-loop approximation [32] in the following form:

$$\begin{aligned}\nu_p^{-1} = \sigma - \frac{(n+2)}{n+8}\varepsilon + \frac{(n+2)(7n+20)\alpha_{S,0}}{(n+8)^3}\varepsilon^2 \\ + \frac{(n+2)\varepsilon^3}{(n+8)^5}\Big[-4(5n+22)(7n+20)\,\alpha_{S,0}^2 \\ + (n+8)^2(7n+20)(\alpha_{S,1} - 2\alpha_{D,1}\alpha_{S,0}) \\ + (n+8)(-8(n-1)\alpha_T + 2(n^2+20n+60)\,\alpha_U \\ + 2(n^2+24n+56)\alpha_{I_1} + (5n^2+28n+48)\alpha_{I_2} \\ + (5n+22)\alpha_{I_4})\Big] + O(\varepsilon^4),\end{aligned} \qquad (8)$$

where α_K with $K = \{S, D, I_1, I_2, I_4, T, U\}$ are expressed in terms of the loop integrals (for details see [32]). $\alpha_{S,i}$ and $\alpha_{D,i}$ are the coefficients at ε^i in the ε-expansion series of α_S and α_D. Explicit expressions for α_K are given in the Appendix. We get the following expression for n_c:

$$\begin{aligned} n_c(d,\sigma) = 4 + 4\alpha_{S,0}\varepsilon + \Big(56\alpha_{I_1} + 40\alpha_{I_2} + 7\alpha_{I_4} \\ -192\alpha_{D,1}\alpha_{S,0} - 56\alpha_{S,0}^2 + 96\alpha_{S,1} - 4\alpha_T + 52\alpha_U\Big)\frac{\varepsilon^2}{24} + O(\varepsilon^3).\end{aligned} \qquad (9)$$

Taking into account the explicit expression for $\alpha_{S,0}$, one can check that up to the first order of ε in Eq. (9) coincides with the two-loop result (6).

Formally, the numerical value of n_c at fixed d and σ can be calculated from the expansion (9) by using expressions for α_K from the Appendix, recalling that $\varepsilon = 2\sigma - d$ and substituting the values of d and σ. However, the ε-expansions are known to be asymptotic at best [7]. Therefore, one has to apply special resummation procedures to restore their convergence in order to get reliable numerical estimates on their basis. Doing so, we start our analysis by representing series (9) by means of the diagonal Padé approximant $[1/1](\varepsilon)$ [41]. The result for n_c as a function of σ is shown by dashed lines in Figs. 1(a) and 1(b) for fixed $d = 2$ and $d = 3$, correspondingly. As noted in the previous section, in the region of n and σ above the lines, the critical behavior of the diluted model is the same as for an undiluted model with long-range interactions. For the values of n and σ below the lines, the new disorder long-range universality class is induced. It is known that in the short-range case, the Padé approximants of the three-loop expansion give values of n_c that exceed the most accurate estimates [27]. It seems to be also in the long-range case, since the data of the NPRG approach (shown by black dots in Fig. 1) are located below the lines calculated via Padé approximant*. To enhance our estimates, we use also a more elaborated Padé–Borel resummation technique [42]. First, to weaken the factorial growth of the expansion coefficients, we write the Borel transform for (9) as

* These data were obtained from interpolation of the NPRG results for critical exponents of the long-range n-vector model [15].

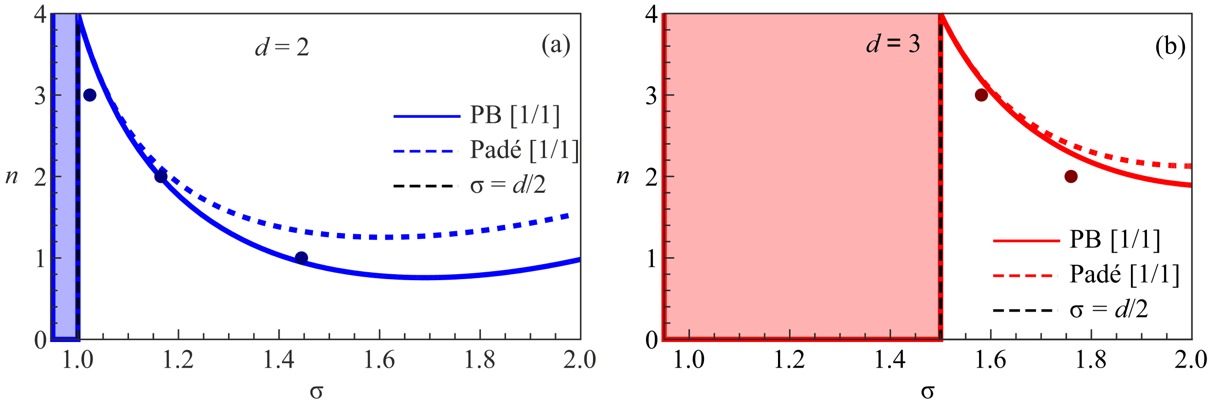

Fig. 1. (Color online) Resummed values of $n_c(\sigma)$ obtained by the Padé approximant [1/1] (dashed lines) and Padé–Borel resummation (solid lines) for d = 2 (a) and 3 (b). For the region of values n and σ above the lines, the pure long-range universality class holds, while the new disorder long-range universality class is induced for the values below the lines. Dots show results that follow from the interpolation of the NPRG data of Ref. 15. The mean-field behavior holds for $\sigma < d / 2$ (regions separated by vertical dashed lines).

$$\sum_{k=0}^{2} n_k \varepsilon^k \to \sum_{k=0}^{2} \frac{n_k \varepsilon^k}{k!}. \qquad (10)$$

An analytical continuation of the Borel transform is achieved by representing it in the form of the diagonal Padé approximant $[1/1]_B(\varepsilon)$, where subscript B is used to distinguish from the Padé approximant of the original series (9). Finally, the resummed function is obtained via an inverse Borel transform:

$$n_c^{\text{res}}(\varepsilon) = \int_0^\infty dt\, e^{-t}[1/1]_B(\varepsilon t). \qquad (11)$$

Results following from the Padé–Borel resummation are presented in Fig. 1 by solid lines for d = 2 and 3. For d = 3 , the Padé–Borel resummation leads to lower values of $n_c(\sigma)$, as compared to those obtained form the [1/1] Padé approximant. This is a right tendency, as is seen also from comparing our results to the NPRG data shown by dots in Fig. 1. As usual with the perturbative expansions, the accuracy of the results decreases with an increase in the expansion parameter, in our case it is $\varepsilon = 2\sigma - d$. Therefore, our results are less accurate for $d = 2$, where the expansion parameter changes within $0 \le \varepsilon \le 2$ for $1 \le \sigma \le 2$. But even then the results of Padé–Borel resummation may serve as reliable estimates up to the moderate values of $\sigma \simeq 1.5$. A remarkable feature of the plots presented in Figs. 1(a) and 1(b) is that for a certain range of parameters σ there are regions in the (n, σ) plane that correspond to integer values of $n = 1, 2, 3$ and lie below the $n_c(\sigma)$ curve. This means that the new disorder long-range universality class is induced in the n-vector model not only for the Ising ($n = 1$), but also for the XY ($n = 2$) and classical Heisenberg (n = 3) cases.

5. Conclusions

In this study, we were interested in the question how the critical behavior of a many-particle system is changed under the competing influence of two factors: type of interaction and structural disorder? To this end, we have considered the original model to describe criticality, an n-vector spin model (1), and analyzed changes in its critical behavior provided the interaction between spins is of a long-range nature (2) and an underlying lattice structure is disordered. To be more specific, we considered the case when the weak quenched structural disorder leads to fluctuations in the local transition temperature (5). The literature available so far [22, 23] reported that second-order phase transition in such a model can belong to the new, disorder induced long-range universality class. However, such a qualitative conclusion has to be supported by quantitative estimates of the region of model parameters where the new universality class can manifest.

To do so, we have calculated the marginal dimension $n_c(\sigma)$ of the structurally-disordered long-range interacting n-vector model. For given space dimension d and interaction decay σ, the model with the order parameter component number $n < n_c$ belongs to the new disorder-induced long-range universality class. Based on the recent results for the critical exponents of the pure long-range n-vector model [32], we used the Harris criterion to calculate $n_c(\sigma)$ with the record three-loop accuracy, Eq. (9). This enabled us to apply familiar resummation techniques to evaluate numerical values of the marginal dimension, as shown in Figs. 1(a) and 1(b) for space dimensions d = 2 and d = 3. Obtained results serve as a solid argument that not only the Ising-like (n = 1) systems, but also systems that are described by the XY (n = 2) and Heisenberg (n = 3) models belong to the new universality class for the moderate values of σ at space dimensions d = 2 and 3.

Acknowledgments

We thank the Editors, Yurij Naidyuk, Larissa Brizhik, and Oleksandr Kovalev, for their invitation to contribute to the Festschrift in memory of Oleksandr Davydov. This work was supported in part by the grant of the National Academy of Sciences of Ukraine for research laboratories/groups of young scientists No. 07/01-2022(4) (D. S.) and by the U. S. Department of Energy (DOE), Office of Science, Basic Energy Sciences, Materials Science and Engineering Division, under Award No. DE-SC0013599 (M. D. ans D. S.). M. D. thanks members of LPTMC for their hospitality during his academic visit in the Sorbonne University, where the part of this work was done.

Appendix

In this Appendix, we list expressions for α_K, as they were given in Ref. 32:

$$\alpha_D = 1 + \frac{\varepsilon}{2}\left[\psi(1) - \psi\left(\frac{d}{2}\right)\right] + \frac{\varepsilon^2}{8}\left[\left(\psi(1) - \psi\left(\frac{d}{2}\right)\right)^2 + \psi_1(1) - \psi_1\left(\frac{d}{2}\right)\right],$$

$$\alpha_S = 2\psi\left(\frac{d}{4}\right) - \psi\left(\frac{d}{2}\right) - \psi(1) + \frac{\varepsilon}{4}\left[\left[2\psi\left(\frac{d}{4}\right) - \psi\left(\frac{d}{2}\right) - \psi(1)\right] \times \left[3\psi(1) - 5\psi\left(\frac{d}{2}\right) + 2\psi\left(\frac{d}{4}\right)\right] + 3\psi_1(1) + 4\psi_1\left(\frac{d}{4}\right) - 7\psi_1\left(\frac{d}{2}\right) - 4J_0\left(\frac{d}{4}\right)\right],$$

$$\alpha_U = \alpha_{I_2} = -\psi_1(1) - \psi_1\left(\frac{d}{4}\right) + 2\psi_1\left(\frac{d}{2}\right) + J_0\left(\frac{d}{4}\right),$$

$$\alpha_T = \frac{1}{2}\left[2\psi\left(\frac{d}{4}\right) - \psi\left(\frac{d}{2}\right) - \psi(1)\right]^2 + \frac{1}{2}\psi_1(1) + \psi_1\left(\frac{d}{4}\right) - \frac{3}{2}\psi_1\left(\frac{d}{2}\right) - J_0\left(\frac{d}{4}\right),$$

$$\alpha_{I_1} = \frac{3}{2}\left[2\psi\left(\frac{d}{4}\right) - \psi\left(\frac{d}{2}\right) - \psi(1)\right]^2 + \frac{1}{2}\psi_1(1) - \frac{1}{2}\psi_1\left(\frac{d}{2}\right),$$

$$\alpha_{I_4} = 6\frac{\Gamma\left(1+\frac{d}{4}\right)^3\Gamma\left(-\frac{d}{4}\right)}{\Gamma\left(\frac{d}{2}\right)}\left[\psi_1(1) - \psi_1\left(\frac{d}{4}\right)\right]. \tag{A.1}$$

In the above expressions, ψ_i are the polygamma functions of order i, while J_0 is the following sum:

$$J_0\left(\frac{d}{4}\right) = \frac{1}{\Gamma\left(\frac{d}{4}\right)^2}\sum_{n\geq 1}\frac{\Gamma\left(n+\frac{d}{2}\right)\Gamma\left(n+\frac{d}{4}\right)^2}{n(n!)\Gamma\left(\frac{d}{2}+2n\right)} \times\left[2\psi(n+1) - \psi(n) - 2\psi\left(n+\frac{d}{4}\right) - \psi\left(n+\frac{d}{2}\right) + 2\psi\left(\frac{d}{2}+2n\right)\right]. \tag{A.2}$$

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Про новий клас універсальності у структурно-невпорядкованій *n*-векторній моделі з далекосяжними взаємодіями

Dmytro Shapoval, Maxym Dudka, Yurij Holovatch

Досліджено межу стійкості області, де нетривіальна критична поведінка *n*-векторної моделі з далекосяжними степенево загасаючими взаємодіями зумовлюється наявністю структурного безладу (наприклад, слабке заморожене розведення). Ця межа задається маргінальною вимірністю параметра порядку n_c, що залежить від вимірності простору d та контролюючого параметру загасання взаємодії σ, нижче якої модель належить до нового класу універсальності, зумовленого розведенням. Використовуючи критерій Гарріса та нещодавні результати теоретико-польової ренормгрупи для чистої моделі з далекосяжними взаємодіями, ми отримали n_c у вигляді розкладу за $\varepsilon = 2\sigma - d$ у трипетлевому наближенні. Розраховано числові значення для n_c із застосуванням методів пересумовування. Отримані результати показують, що до нового класу універсальності, зумовленого безладом, при $d = 2$ та 3 належать не тільки ізінгівські системи ($n = 1$).

Ключові слова: далекосяжні взаємодії, заморожений безлад, ренормгрупа, маргінальна вимірність.